# Effect of Tap Spacing on the Performance of Direct-Sequence Spread-Spectrum RAKE Receiver

Ki Jun Kim, *Member, IEEE*, Soon Yil Kwon, *Member, IEEE*, Een Kee Hong, *Member, IEEE*, and Keum Chan Whang, *Member, IEEE*

***Abstract*—In this paper, the effect of tap spacing on the performance of a RAKE receiver is analyzed analytically in a frequency-selective fading channel. A continuous-time multipath fading channel model is used for the analysis, and the expression of correlation between the desired signals, interference signals, and noise signals at the output of each branch of the RAKE receiver is derived for various chip waveforms. Since the noise components of each branch signal are correlated to each other, an optimum combining rule based on the maximum-likelihood criterion is derived to gain utmost performance. It is shown that the performance of the system can be improved by setting the tap spacing of the RAKE receiver below the chip duration when the bandwidth of the transmitted signal is larger than the inverse of the chip duration. Also, it is shown that the normalized capacity of the system can be increased by using a chip waveform occupying wider bandwidth, which takes advantage of the increased diversity gain merits of a wide-band code-division multiple-access system at the same chip rate. It is noted that the derived combining rule gives diversity gain against the fading process as well as noise whitening processing gain against multiple-access interference at the same time.**

***Index Terms*—Code-division multiple access, communication system performance, diversity methods, maximum-likelihood detection, multipath channels.**

## I. Introduction

DIRECT-SEQUENCE code-division multiple-access (DS-CDMA) communication systems have recently attracted considerable attention as a mobile cellular and a IMT-2000 communication system by reason of its ability to suppress a wide variety of interfering signals including narrow-band interference, multiple-access interference (MAI), and multipath interference (MPI). In the presence of frequency-selective fading, the capacity of the system can be enormously enlarged through multipath diversity gained by utilizing a RAKE receiver structure [1], [2].

There has been many researches on the performance of the RAKE receiver using different channel models, system models, and analytical techniques. A discrete-time multipath channel model was used in [3]–[6], where it is assumed that each multipath component is spaced wider than the chip duration so that the signals received at each tap are independent from one another. A tap delay line (TDL) channel model was used in [7]–[9] by assuming that the bandwidth of the transmitted signal is limited within the inverse of the chip duration. The TDL channel model is improper for cases in which the bandwidth of the transmitted signal is wider than the inverse of chip duration or when the signal is time limited rather than bandwidth limited. Thus, in [10], a continuous-time multipath channel model was used to evaluate the performance of the RAKE receiver considering the correlation between the signals received at each tap. But, only the cases when the interval between the taps in the RAKE receiver is integer times the chip duration are considered.

In this paper, we evaluate the effect of tap spacing on the RAKE receiver, particularly when the tap spacing is not integer times the chip duration. For this purpose, the expression of correlation between the desired signals, interference signals, and noise signals at the output of each branch of the RAKE receiver is derived. Also, since the commonly used maximum-ratio combining (MRC) is no longer optimum when the noise components of each branch signal are correlated to each other, we investigate the optimum combining rule based on the maximum-likelihood (ML) criterion.

A further issue of this paper is to evaluate the effect of chip waveform on the RAKE receiver in relation to the tap spacing. In [11], it is shown that the capacity of a CDMA system can be maximized by using a chip waveform with a rectangular-shaped spectrum in a Gaussian channel. A raised-cosine pulse with a rolloff factor nearly equal to zero is used as the chip waveform in systems based on IS-95 [12]. However, in a frequency-selective fading channel, it is conceptually possible to achieve more multipath (frequency) diversity gain by using chip waveforms with larger rolloff factors, which increases the amount of bandwidth used without increasing the chip rate of the system. But, if the tap spacing is integer times the chip duration and the number of taps in the RAKE receiver is fixed, it is not possible to achieve more diversity gain by only increasing the rolloff factor. As a way to achieve such diversity gain, a RAKE receiver that uses more taps with reduced tap spacing can be considered. Thus, bit-error rates (BER's) of the RAKE receiver in terms of tap spacing for various chip waveforms are obtained analytically by exploiting the characteristic function of the decision variable, based on random signature sequences and Gaussian approximation.

Section II presents the DS-CDMA system and channel model to be considered. In Section III, the optimum combining rule is

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derived from the ML criterion. The performance of the RAKE receiver is analyzed in terms of BER in Section IV. The BER and system capacity results are shown in Section V according to the RAKE receiver tap spacing for a rectangular waveform and a raised-cosine waveform. Finally, conclusions are made in Section VI.

## II. System and Channel Model

The DS-CDMA system to be considered consists of $K$ simultaneous users. The data sequence of each user is modulated by the unique signature sequence, such that $N$ continuous chips are modulated by one data bit. Here, $N$ is the processing gain of the system defined as the ratio of symbol duration $T$ and chip duration $T_c$. For the reverse link of the system, the low-pass equivalent BPSK data modulated transmitted signal of the $k$th user is given by

$$s_k(t) = \sqrt{2E_b/N} \sum_{n=-\infty}^{\infty} d_{k,n} h(t - nT_c) \tag{1}$$

where $E_b$ is the signal energy per bit and $h(t)$ denotes the chip waveform that is normalized such that $\int_{-\infty}^{\infty} |h(t)|^2 \, dt = 1$. $d_{k,n}$ is the data modulated $n$th chip of the $k$th user and is defined as

$$d_{k,n} = b_{k,\lfloor n/N \rfloor} a_{k,n} \tag{2}$$

where $b_{k,n}$ and $a_{k,n}$ are the data and the signature sequences of the $k$th user, respectively. Both are modeled as independent random sequences taking on the values $+1$ and $-1$ with equal probability. $\lfloor x \rfloor$ denotes the largest integer not greater than $x$.

The mobile communication channel is modeled as a wide-sense stationary uncorrelated scattering (WSSUS) frequency-selective Rayleigh fading channel. Then, the signal received from the $k$th user is [10], [13]

$$r_k(t) = \int_{-\infty}^{\infty} \beta_k(\varepsilon) s_k(t - \varepsilon) \, d\varepsilon = \beta_k(t) \otimes s_k(t) \tag{3}$$

where $\otimes$ denotes the convolutional operation. $\beta_k(t)$ is the channel impulse response of the $k$th user's link, which is modeled as a complex zero-mean Gaussian random process and has an autocorrelation function as follows:

$$E\{\beta_k(t_1)\beta_k^*(t_2)\} = g(t_1)\delta(t_1 - t_2) \tag{4}$$

where $\delta(\cdot)$ denotes the Dirac delta function. The function $g(t)$ is called the multipath intensity profile (MIP), which gives the average power output of the channel as a function of the time delay $t$ and is normalized to $\int g(t) \, dt = 1$. Note that by normalizing $g(t)$, $E_b$ in (1) can be interpreted as the average total received energy per bit that will be assumed to be equal for all users for simplicity.

The total received signal at the receiver front end can be written as

$$r(t) = \sum_{k=1}^{K} \beta_k(t) \otimes s_k(t - \tau_k) + n(t) \tag{5}$$

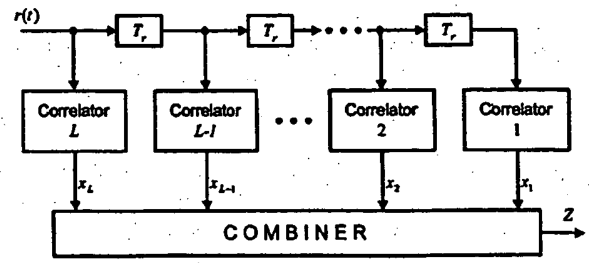


Fig. 1. RAKE receiver model.

where $n(t)$ is a low-pass equivalent process of the additive white Gaussian noise with double-sided power spectrum density $N_0/2$.

At the receiver matched to the first user's signature sequence, the received signal of (5) is passed through a TDL with tap spacing $T_r$, as shown in Fig. 1. At each tap, the received signal is despread by passing the signal through a correlator matched to the first user's spreading sequence. Note that the outputs of the correlators are identical to the outputs of a filter matched to the first user's spreading sequence sampled at $T_r$. The filter output can be written as

$$X(t) = \sum_{k=1}^{K} \beta_k(t) \otimes s_k(t - \tau_k) \otimes v(t) + n(t) \otimes v(t) \tag{6}$$

where $v(t) = \sqrt{1/N} \sum_{n=0}^{N-1} a_{1,n}^* h^*(-t - nT_c)$. It is assumed that the receiver knows the exact timing of the first user's signal, i.e., $\tau_1 = 0$. By performing the convolution of $v(t)$ and $s_k(t)$ in (6), this can be simplified as follows:

$$X(t) = X_d(t) + X_s(t) + \sum_{k=2}^{K} X_k(t) + X_n(t) \tag{7}$$

where

$$X_d(t) = \sqrt{2E_b}\, b_{1,0} \beta_1(t) \otimes R_{hh}(t) \tag{8}$$

$$X_s(t) = \sqrt{2E_b} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \beta_1(t) \otimes c_{1,n} R_{hh}(t - nT_c) \tag{9}$$

$$X_k(t) = \sqrt{2E_b} \sum_{n=-\infty}^{\infty} \beta_k(t) \otimes c_{k,n} R_{hh}(t - nT_c - \tau_k) \tag{10}$$

$$X_n(t) = n(t) \otimes v(t) \tag{11}$$

$$c_{k,n} = \frac{1}{N} \sum_{m=0}^{N-1} b_{k,\lfloor (m+n)/N \rfloor} a_{k,m+n} a_{1,m}^* \tag{12}$$

$$R_{hh}(t) = \int_{-\infty}^{\infty} h(\varepsilon) h^*(\varepsilon - t) \, d\varepsilon. \tag{13}$$

Here, $X_d(t)$, $X_s(t)$, $X_k(t)$, and $X_n(t)$ each denote the desired signal component of the first user, the MPI component, the MAI component due to the $k$th user, and the Gaussian noise component, respectively. $c_{k,n}$ of (12) represents the discrete cross correlation function between the first and the $k$th user. If the period of the signature sequence is assumed to be much greater than the processing gain $N$, so that the signature sequence has a characteristic similar to a random sequence, $c_{1,n}$'s except $c_{1,0}$ can be

modeled as zero-mean binomially distributed random variables with autocorrelation as follows:

$$E\{c_{1,m}c_{1,n}^*\} = \begin{cases} 1/N, & \text{if } m=n\neq 0 \\ 1/N-|n|/N^2, & \text{if } m=-n\neq 0,\ |n|<N \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

Also, $c_{k,n}$, $(k \neq 1)$ can be modeled as independent random variables binomially distributed with zero mean and variance equal to $1/N$. $R_{hh}(t)$ of (13) represents the autocorrelation function of the chip waveform.

The receiver is assumed to have complete knowledge of the channel response of the first user including the chip waveform shaping filters in the transmitter and receiver. The channel response of the first user is defined as

$$Y(t) = \sqrt{2E_b}\,\beta_1(t) \otimes R_{hh}(t). \quad (15)$$

It is noted that (15) is identical to (8) except for the fact that the data component exists in (8).

For the RAKE receiver using MRC, which will be denoted as the conventional RAKE receiver hereafter, the decision variable is obtained by summing the $L$ branch signals as

$$Z = \sum_{l=1}^{L} x_l y_l^* + x_l^* y_l \quad (16)$$

where $x_l, y_l$ is defined as $x_l = X((l-1)T_r), y_l = Y((l-1)T_r)$, respectively.

Since $x_l$'s are correlated as shown in the Appendix, the above combining rule is no longer optimum. In the following section, an optimum rule to combine the energy in each branch of the RAKE receiver is derived based on the ML criterion.

## III. Optimun Combining Rule

Assuming that the receiver has exact knowledge of the amplitude and phase of the desired signal, the likelihood ratio is formed as

$$\Lambda(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{x}|b_{1,0}=+1, \mathbf{y})}{p(\mathbf{x}|b_{1,0}=-1, \mathbf{y})} \quad (17)$$

where $p(\mathbf{x}|b_{1,0}, \mathbf{y})$ is the conditional joint density of the signal vector $\mathbf{x} = [x_1, x_2, \cdots, x_L]$ given $b_{1,0}$ and $\mathbf{y} = [y_1, y_2, \cdots, y_L]$. The conditional mean of $\mathbf{x}$ is $b_{1,0} \cdot \mathbf{y}$. Assuming that $\mathbf{x}$ is a Gaussian random vector, the log-likelihood ratio is given by

$$\log\{\Lambda(\mathbf{x}|\mathbf{y})\} = (\mathbf{x}+\mathbf{y})\mathbf{C}_{\mathbf{xx}}^{-1}(\mathbf{x}+\mathbf{y})^{*T} - (\mathbf{x}-\mathbf{y})\mathbf{C}_{\mathbf{xx}}^{-1}(\mathbf{x}-\mathbf{y})^{*T} \quad (18)$$

where $\mathbf{C}_{\mathbf{xx}}$ is the conditional autocovariance matrix of $\mathbf{x}$ given $b_{1,0}$ and $\mathbf{y}$. By the definition of $\mathbf{x}$, the elements of $\mathbf{C}_{\mathbf{xx}}$ are determined by the conditional autocovariance function of $X(t)$ given $b_{1,0}$ and $Y(t)$, which are given in the Appendix. Since $\mathbf{C}_{\mathbf{xx}}$ is a nonnegative definite Hermitian matrix, it is possible to factorize $\mathbf{C}_{\mathbf{xx}}$ through simple linear algebra as follows:

$$\mathbf{C}_{\mathbf{xx}} = \mathbf{U}_{\mathbf{xx}}\mathbf{D}_{\mathbf{xx}}\mathbf{U}_{\mathbf{xx}}^{*T} \quad (19)$$

where $\mathbf{U}_{\mathbf{xx}}$ is a unitary matrix with $L$ eigenvectors of $\mathbf{C}_{\mathbf{xx}}$ as its columns, and $\mathbf{D}_{\mathbf{xx}}$ is a diagonal matrix with $L$ eigenvalues of $\mathbf{C}_{\mathbf{xx}}$ as its diagonal elements in the same order as the columns of $\mathbf{U}_{\mathbf{xx}}$. Note that the eigenvalues of $\mathbf{C}_{\mathbf{xx}}$ are greater than or equal to zero. Using these notations, (18) can be written as

$$\begin{aligned} \log\{\Lambda(\mathbf{x}|\mathbf{y})\} &= (\mathbf{x}'+\mathbf{y}')(\mathbf{x}'+\mathbf{y}')^{*T} - (\mathbf{x}'-\mathbf{y}')(\mathbf{x}'-\mathbf{y}')^{*T} \\ \mathbf{x}' &= [x_1', x_2', \cdots, x_L'] = \mathbf{x}\mathbf{U}_{\mathbf{xx}}\sqrt{\mathbf{D}_{\mathbf{xx}}^{-1}} \\ \mathbf{y}' &= [y_1', y_2', \cdots, y_L'] = \mathbf{y}\mathbf{U}_{\mathbf{xx}}\sqrt{\mathbf{D}_{\mathbf{xx}}^{-1}}. \end{aligned} \quad (20)$$

Due to the transformation of (20), the elements of transformed vector $\mathbf{x}'$ are independent Gaussian random variables. The value of $\mathbf{y}' \cdot \mathbf{y}'^T$ is the total real energy of the received signal. The final decision variable of the RAKE receiver using a combining rule based on the ML criterion, which will be denoted as the optimum RAKE receiver hereafter, is given by

$$Z = \log\{\Lambda(\mathbf{x}|\mathbf{y})\} = \sum_{l=1}^{L} x_l' y_l'^* + x_l'^* y_l'. \quad (21)$$

Note that if the elements of $\mathbf{x}$ are independent random variables, $\mathbf{D}_{\mathbf{xx}}$ and $\mathbf{U}_{\mathbf{xx}}$ are diagonal matrixes. In this case, (21) is identical to (16). As will be treated in the next section, the elements of $\mathbf{x}$ are correlated for most cases of chip waveforms and tap spacings. Thus, the combining rule of (21) must be used instead of (16) to gain better performance.

## IV. Perfomance Analysis

The performance of the RAKE receivers based on the conventional and the optimum combining rule are analyzed in terms of probability of error, defined as the probability that the decision variable is less than zero when the transmitted data is +1. Since $x_l$ and $y_l$ can be approximated as complex Gaussian processes by the central limit theorem [4]–[9], the decision variable $Z$ can be written in a Hermitian quadratic form of the complex Gaussian random vector $\mathbf{z} = [\mathbf{x}\ \mathbf{y}]$ as

conventional RAKE receiver:

$$Z = \mathbf{z}^*\mathbf{Q}\mathbf{z}^T$$

optimum RAKE receiver:

$$\begin{aligned} Z &= \mathbf{z}'^*\mathbf{Q}\mathbf{z}'^T \\ &= \mathbf{z}^*\mathbf{U}_{\mathbf{zz}}^*\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}}^*\mathbf{Q}\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}}^T\mathbf{U}_{\mathbf{zz}}^T\mathbf{z}^T. \end{aligned}$$

Here, $\mathbf{Q}$, $\mathbf{U}_{\mathbf{zz}}$, and $\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}}$ are $(2L \times 2L)$ matrixes defined as

$$\mathbf{Q} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{I} & 0 \end{bmatrix} \quad (22)$$

$$\mathbf{U}_{\mathbf{zz}} = \begin{bmatrix} \mathbf{U}_{\mathbf{xx}} & 0 \\ 0 & \mathbf{U}_{\mathbf{xx}} \end{bmatrix} \quad (23)$$

$$\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}} = \begin{bmatrix} \sqrt{\mathbf{D}_{\mathbf{xx}}^{-1}} & 0 \\ 0 & \sqrt{\mathbf{D}_{\mathbf{xx}}^{-1}} \end{bmatrix} \quad (24)$$

where $\mathbf{I}$ is an $(L \times L)$ identity matrix.

The probability of error can be obtained by using the characteristic function of the decision variable as in [10] and [14].

$$P_e = \sum_{\lambda_i<0} \prod_{\substack{l=1 \\ l\neq i}}^{2L} \frac{1}{1-\lambda_l/\lambda_i}. \tag{25}$$

Here, $\{\lambda_i, i = 1, 2, \cdots, 2L\}$ are the eigenvalues of $\mathbf{R}\cdot\mathbf{Q}$ for the conventional RAKE receiver and those of $\mathbf{R}\mathbf{U}_{\mathbf{zz}}^{*}\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}}^{*T}\mathbf{Q}\sqrt{\mathbf{D}_{\mathbf{zz}}^{-1}}\,\mathbf{U}_{\mathbf{zz}}^{T}$ for the optimum RAKE receiver. $\mathbf{R} = [R_{i,j}] = E\{\mathbf{z}^T \cdot \mathbf{z}^*\}$ is the conditional autocorrelation matrix of vector $\mathbf{z}$ given $b_{1,0} = 1$. $\mathbf{R}$ is determined by the autocorrelation and cross correlation of $X(t)$ and $Y(t)$, which are shown in the Appendix.

## V. NUMERICAL RESULTS

This section shows the effects of tap spacing and the chip waveform on the performance of the conventional and the optimum RAKE receivers based on the analysis method in the previous section. The waveforms to be considered are a time-limited rectangular pulse as in (26) and a frequency-limited square-root raised-cosine pulse as in (27) [15].

$$h(t) = \begin{cases} 1/\sqrt{T_c}, & \text{if } 0 \le t \le T_c \\ 0, & \text{otherwise} \end{cases} \tag{26}$$

$$h(t) = 4\rho \frac{\cos[(1+\rho)\pi t/T_c] + \sin[(1-\rho)\pi t/T_c]/(4\rho t/T_c)}{\pi\sqrt{T_c}\,[(4\rho t/T_c)^2 - 1]}. \tag{27}$$

Rolloff factors of $\rho = 0, 0.5, 1.0$ are considered for the raised-cosine pulse.

Considering two correlators which are matched to multipaths with delay time $t_1$ and $t_2$, respectively, the correlation between the outputs consists of components of the desired signal and total interference (i.e., MPI, MAI, and white Gaussian noise) as defined in (A2). The correlation between the desired signal decreases the diversity gain of the RAKE receiver, and the correlation between the total interference decreases the total combined average signal to interference ratio of the RAKE receiver. Moreover, the correlation between the total interference makes MRC no longer optimum. In a practical system accommodating a lot of users, the effect of MPI is negligible. In such a system, MAI is dominant among the total interference when signal-to-noise-ratio (SNR) is high, while Gaussian noise is dominant when SNR is low. The correlation coefficients of the correlator outputs due to MAI and Gaussian noise are shown in Fig. 2 as a function of normalized time difference. From the figure, it is seen that for the general case of the RAKE receiver having a tap spacing of $T_c$, there is no correlation due to Gaussian noise. However, correlation due to MAI exists except for the case of a raised-cosine pulse with $\rho = 0$, i.e., sinc waveform. So, the conventional RAKE receiver is not optimal. It is noted that as the tap spacing is reduced below the chip duration, the increase in the amount of correlation between total interference results in performance degradation. However, if the number of taps in the RAKE receiver is enlarged with a narrower tap spacing over the full range of the delay spread, it is seen that the optimum receiver fully overcomes this degradation by collecting more energy, while the conventional receiver partially overcomes this.

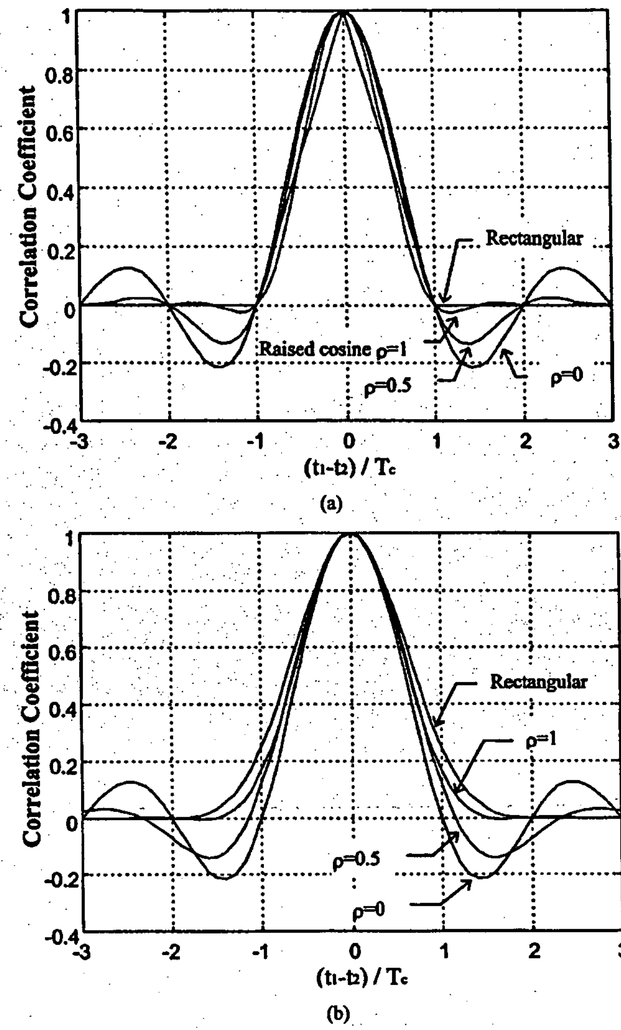


Fig. 2. Correlation coefficient between taps of RAKE receiver. Correlation coefficients due to (a) Gaussian noise and (b) MAI

In order to show the performance of the RAKE receiver according to the tap spacing, the BER of the system using a rectangular chip waveform is denoted in Fig. 3 as a function of average SNR per bit, i.e., $E_b/N_0$. A uniform MIP over $[0, \tau_{\max}]$ is assumed for Fig. 3 and also for the rest of the paper. The maximum delay spread, the processing gain, and the number of simultaneous users are set as $\tau_{\max} = 2T_c$, $N = 256$, and $K = 11$, respectively. The tap spacings considered are equal to $0.7T_c$, $1.0T_c$, and $1.3T_c$. Four taps are located at $0, 0.7, 1.4, 2.1T_c$ when $T_r = 0.7T_c$, three taps are located at $0, 1, 2T_c$ when $T_r = T_c$, and three taps are located at $0, 1.3, 2.6T_c$ when $T_r = 1.3T_c$. Though uniform MIP over $[0, \tau_{\max}]$ is assumed, signal energy may also be found outside the region $[0, \tau_{\max}]$ due to the shape of chip waveform. From (A3), the received signal energy distribution on the time axis can be derived. The received energy for rectangular waveform resides in $[-T_c, \tau_{\max}+T_c]$, although most of the energy resides in $[0, \tau_{\max}]$.

It can be seen in the figure that both the conventional and optimum RAKE receivers show best performance when the tap

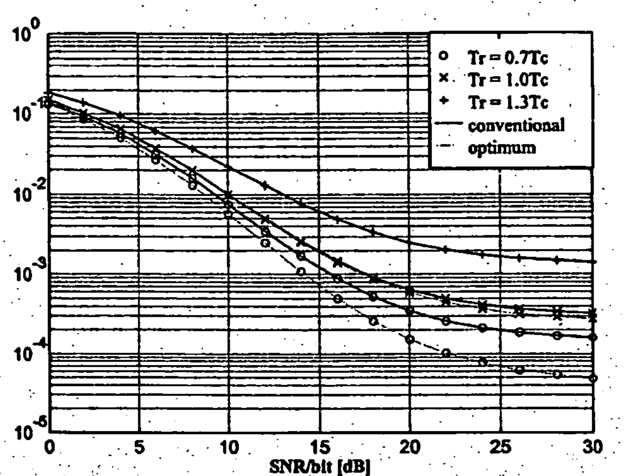


Fig. 3. BER for DS-CDMA system according to tap spacing of the RAKE receiver (rectangular chip waveform, $\tau_{\max} = 2T_c$, $K = 11$, $N = 256$)

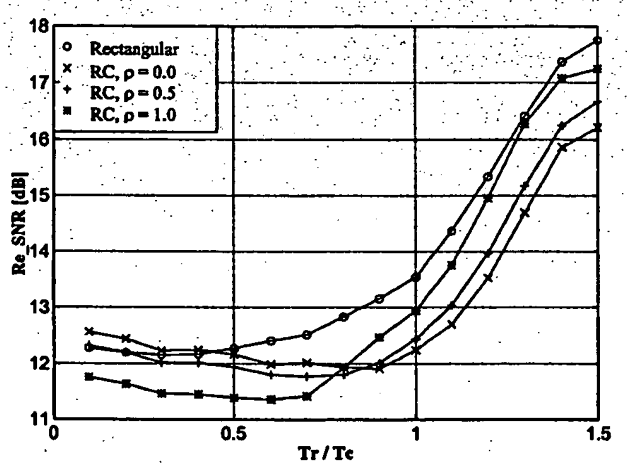


Fig. 4. Required SNR for BER of $10^{-3}$ for DS-CDMA system according to tap spacing of the conventional RAKE receiver ($\tau_{\max} = 2T_c$, $K = 1$, $N = 256$).

spacing is $0.7T_c$. This is because diversity gain is increased by enlarging the number of taps with narrower tap spacing. When $T_r = 1.3T_c$ or $T_r = T_c$, the performance difference between the conventional and optimum RAKE receivers is indistinguishable. This means that the performance degradation of conventional RAKE due to the correlation between outputs of taps is negligible for tap spacing greater than or equal to chip duration. But, when $T_r = 0.7T_c$, the conventional RAKE receiver is inferior to the optimum RAKE receiver because the performance of the former degrades due to the existence of correlation between total interference.

In order to show the performance of the conventional RAKE receiver according to the tap spacing and various waveforms, Fig. 4 depicts the required SNR to achieve a BER of $10^{-3}$ as a function of tap spacing. Here, $\tau_{\max} = 2T_c$, $N = 256$, and one user is considered in order to observe the exclusive effect of Gaussian noise. Also, in order to observe the exclusive effect of MAI, the maximum capacities of the system that satisfy a BER of $10^{-3}$ are shown in Fig. 5 when the Gaussian noise approaches zero. The number of taps is set as $1 + \lceil \tau_{\max}/T_r \rceil$

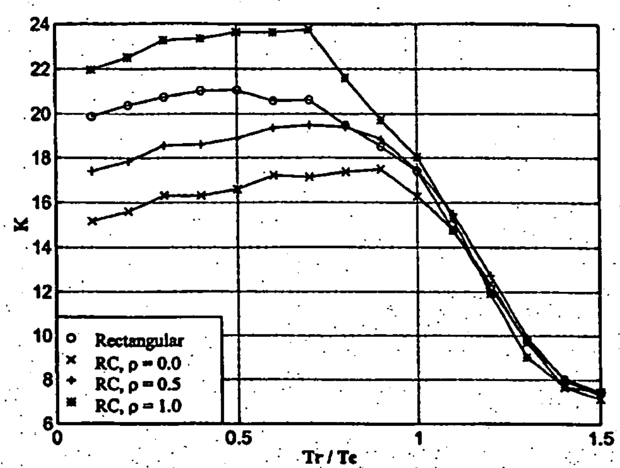


Fig. 5. Capacity of DS-CDMA system according to the tap spacing of the conventional RAKE receiver (BER $= 10^{-3}$, $\tau_{\max} = 2T_c$, $N = 256$, SNR $= \infty$).

for each tap distribution. Here, $\lceil x \rceil$ denotes the smallest integer greater than or equal to $x$. From Figs. 4 and 5, it can be noted that, except for the case of perfect sinc waveform, the system performance improves by setting the tap spacing of the conventional RAKE receiver at a value below the chip duration. It is also noted that performance degrades dramatically as the tap spacing is set above the chip duration. This is because more energy can be gathered from the received signal and also better diversity effect can be achieved as more taps are used with reduced tap spacing. Thus, this means that these effects can overcome the performance degradation due to increase in the amount of correlation as the tap spacing is reduced to a certain point. As the tap spacing is further reduced below this point, the performance degradation due to correlation can no longer be overcome, thus there exists an optimum amount of tap spacing for the conventional RAKE to gain best performance. It is shown that the optimum point decreases as the absolute bandwidth of the chip waveform increases. Representatively, for a raised-cosine waveform with $\rho = 1.0$, the decrease of the tap spacing to $0.7T_c$ results in an increase of 30% in system capacity.

Figs. 6 and 7 show the effect of Gaussian noise on the performance of optimum RAKE receiver in terms of required SNR to achieve a BER of $10^{-3}$ according to the tap spacing of the receiver normalized to $T_c$. Figs. 8 and 9 show the effect of MAI on the performance of optimum RAKE receiver. The maximum delay spread of the channel is set as $\tau_{\max} = 2T_c$ for Figs. 6 and 8, $\tau_{\max} = 4T_c$ for Figs. 7 and 9. It is seen that the performance of the optimum RAKE receiver improves continuously as the tap spacing decreases, since the correlation due to the total interference has been eliminated by the receiver. Comparing Fig. 4 with Fig. 6 and Fig. 5 with Fig. 8, it is noted that the improvement in performance of the optimum RAKE receiver is greater than the conventional RAKE receiver. The improvement can be more clearly noted in terms of capacity than that in terms of required SNR. This is since MAI can be modeled as colored Gaussian noise, except for the case of sinc waveforms, and the transform process of the optimum RAKE receiver can be interpreted as a process similar to the noise whitening process in [16] and [17]. Thus, the optimum RAKE achieves diversity gain

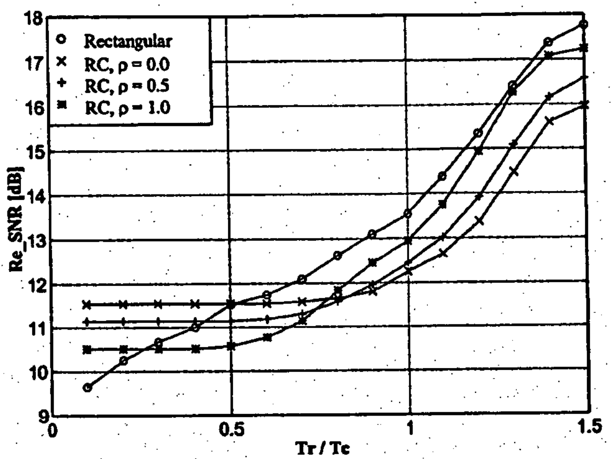


Fig. 6. Required SNR for BER of $10^{-3}$ for DS-CDMA system according to tap spacing of the optimum RAKE receiver ($\tau_{\max} = 2T_c, K = 1, N = 256$).

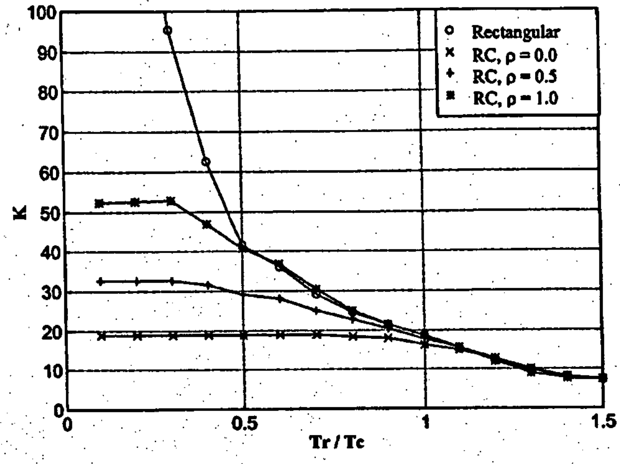


Fig. 8. Capacity of DS-CDMA system according to the tap spacing of the optimum RAKE receiver (BER $= 10^{-3}$, $\tau_{\max} = 2T_c$, $N = 256$, SNR $= \infty$).

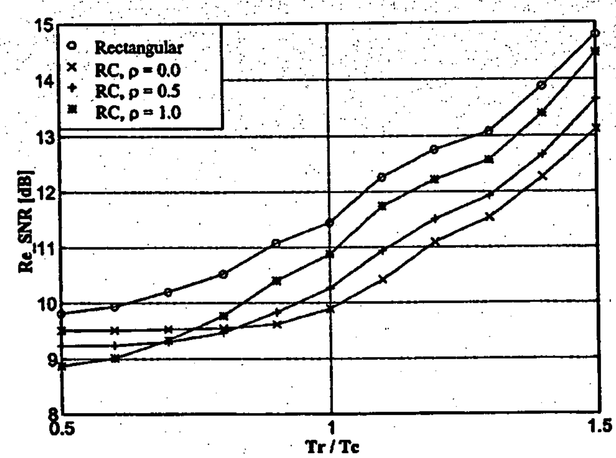


Fig. 7. Required SNR for BER of $10^{-3}$ for DS-CDMA system according to tap spacing of the optimum RAKE receiver ($\tau_{\max} = 4T_c, K = 1, N = 256$).

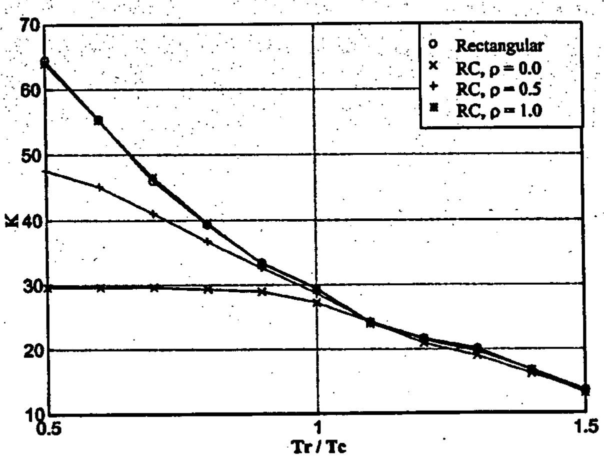


Fig. 9. Capacity of DS-CDMA system according to the tap spacing of the optimum RAKE receiver (BER $= 10^{-3}$, $\tau_{\max} = 4T_c$, $N = 256$, SNR $= \infty$).

against the fading process and noise whitening processing gain against MAI at the same time. Representatively, for a raised-cosine waveform with $\rho = 1.0$, the decrease of the tap spacing to $0.3T_c$ results in the increase of 160% in system capacity.

In order to show the difference in performance according to the chip waveform used, the normalized capacity of DS-CDMA system defined as the total transmission rate per hertz is shown in Fig. 10 for various chip waveforms and spreading bandwidths. The required bandwidth is proportional to $(\rho + 1) \cdot N$. In the figure, by comparing the curves with $\rho = 0$, the performance of the system improves as $N$ is increased from 256 to 512, which shows the well-known fact that wide-band systems can increase capacity by gaining more multipath diversity compared to narrow-band systems. Also, by comparing the curves with $N = 256$, it is noted that performance improves by increasing $\rho$. This means that by using a chip waveform with larger bandwidth, the optimum RAKE receiver can gain the advantage of a wide-band system without increase in processing gain.

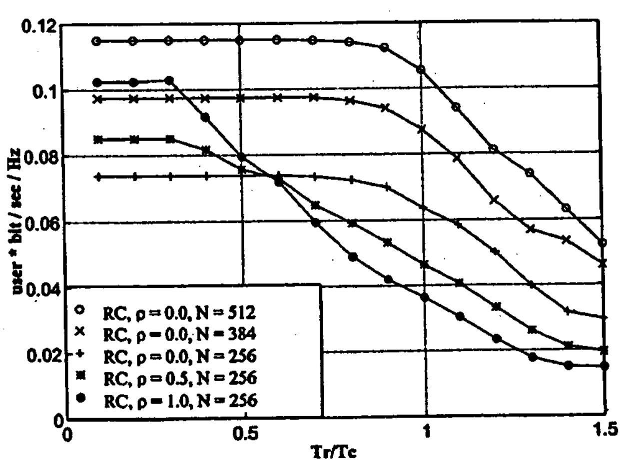


Fig. 10. Normalized capacity of DS-CDMA system according to the tap spacing of the optimum RAKE receiver (BER $= 10^{-3}$, $\tau_{\max} = 2T/256$, SNR $= \infty$).

## VI. Conclusions

The effect of tap spacing and chip waveform on the performance of the RAKE receiver over a continuous-time frequency-selective fading channel has been analyzed analytically. By enlarging the number of taps in the RAKE receiver with a narrower tap spacing over the full range of the delay spread, it is shown that the RAKE receiver can gather more signal energy and achieve more multipath diversity. However, reducing the tap spacing causes an increase in the amount of correlation among the outputs of the taps. In this case, it is shown that the performance of the system can be improved by using a combining rule based on the ML criterion, which can be also thought of as a noise whitening process on MAI. The results show that the performance of the system can be improved by setting the tap spacing of the RAKE receiver below the chip duration when the bandwidth of the transmitted signal is larger than the inverse of the chip duration. Moreover, by using the combining rule based on the ML criterion, it is shown that the normalized capacity of the system can be increased by using a chip waveform occupying wider bandwidth.

## Appendix

This appendix derives the autocovariance and the autocorrelation functions of $X(t)$ that are needed for deriving the optimum combining rule and evaluating BER of the RAKE receiver. Also, the autocorrelation function of $Y(t)$ and the cross correlation function of $X(t)$and $Y(t)$ are given that can be derived through a similar process.

Noting that the four components in (7), i.e., $X_d(t)$, $X_s(t)$, $X_k(t)$, and $X_n(t)$, are all generated from independent sources, the autocovariance function of $X(t)$ conditioned on $b_{1,0}$ is given by

$$\mathrm{cov}\{X(t_1)\cdot X^*(t_2)\} = E\{X_s(t_1)\cdot X_s^*(t_2)\}+\sum_{k=2}^{K} E\{X_k(t_1) \cdot X_k^*(t_2)\}+E\{X_n(t_1)\cdot X_n^*(t_2)\}. \quad \text{(A1)}$$

And, the autocorrelation function of $X(t)$ conditioned on $b_{1,0}$ is given by

$$E\{X(t_1)\cdot X^*(t_2)\} = E\{X_d(t_1)\cdot X_d^*(t_2)\} + E\{X_s(t_1) \cdot X_s^*(t_2)\} + \sum_{k=2}^{K} E\{X_k(t_1)\cdot X_k^*(t_2)\} + E\{X_n(t_1)\cdot X_n^*(t_2)\}. \quad \text{(A2)}$$

Note that (A1) and (A2) are identical except for the first expectation term in the right-hand side of (A2).

The first term in (A2) is due to the desired signal of the first user. Using (8) and considering the stochastic characteristics of $\beta(t)$, this term becomes

$$E\{X_d(t_1)\cdot X_d^*(t_2)\} = 2E_b \int_{-\infty}^{\infty} g_1(\varepsilon)R_{hh}(t_1-\varepsilon)R_{hh}^*(t_2-\varepsilon)\,d\varepsilon. \quad \text{(A3)}$$

The second term in (A2) is due to MPI. Using (9) and the stochastic characteristics of $\beta(t)$, this term becomes

$$E\{X_s(t_1)\cdot X_s^*(t_2)\} = 2E_b \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} E\{c_{1,n}\cdot c_{1,m}^*\} \cdot \int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon-nT_c)R_{hh}^* \cdot (t_2-\varepsilon-mT_c)\,d\varepsilon. \quad \text{(A4)}$$

Using the statistics of $c_{1,n}$ defined in (14), (A4) can be written as

$$\begin{aligned} E\{X_s(t_1)\cdot X_s^*(t_2)\} &= 2E_b/N \sum_{\substack{n=-\infty\\ n\neq 0}}^{\infty} \int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon-nT_c)R_{hh}^* \\ &\quad \cdot (t_2-\varepsilon-nT_c)\,d\varepsilon + 2E_b/N^2 \sum_{\substack{n=-N\\ n\neq 0}}^{N}(N-|n|) \\ &\quad \cdot \int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon+nT_c)R_{hh}^*(t_2-\varepsilon-nT_c)\,d\varepsilon. \end{aligned} \quad \text{(A5)}$$

The third term in (A2) is due to MAI from other users. Using (10) and performing the expectation operation with respect to $\beta(t)$, $c_{k,n}$, and $\tau_k$, this term becomes

$$E\{X_k(t_1)\cdot X_k^*(t_2)\} = \frac{2E_b}{NT_c}\sum_{n=-\infty}^{\infty}\int_0^{T_c}\int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon-\tau_k-nT_c) \cdot R_{hh}^*(t_2-\varepsilon-\tau_k-nT_c)\,d\varepsilon\,d\tau_k. \quad \text{(A6)}$$

Substituting $\tau_k+nT_c$ with $\tau$, and encompassing the summation over $n$ and the integration over $\tau_k$ into a single integration over $\tau$, (A6) can simplified as follows after some manipulations

$$E\{X_k(t_1)\cdot X_k^*(t_2)\} = \frac{2E_b}{NT_c} R_{RR}(t_1-t_2) \quad \text{(A7)}$$

where

$$R_{RR}(t) = \int_{-\infty}^{\infty} R_{hh}(\varepsilon)R_{hh}^*(\varepsilon-t)\,d\varepsilon. \quad \text{(A8)}$$

The last term in (A2) is due to the white Gaussian noise. According to the statistical characteristics of $n(t)$ and $a_{1,n}$, it is easy to show that this term becomes

$$E\{X_n(t_1)\cdot X_n^*(t_2)\} = 2N_0R_{hh}(t_1-t_2). \quad \text{(A9)}$$

Following a method similar to the above, the autocorrelation function of $Y(t)$ and the cross correlation function of $X(t)$and $Y(t)$ is given as

$$E\{Y(t_1)\cdot Y^*(t_2)\} = 2E_b\int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon)R_{hh}^*(t_2-\varepsilon)\,d\varepsilon \quad \text{(A10)}$$

$$E\{X(t_1)\cdot Y^*(t_2)\} = 2E_b\int_{-\infty}^{\infty} g(\varepsilon)R_{hh}(t_1-\varepsilon)R_{hh}^*(t_2-\varepsilon)\,d\varepsilon. \quad \text{(A11)}$$

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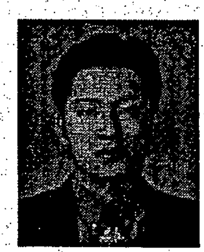

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